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NOTE ON SECOND SOLUTION OF PROBLEM 49, MECHANICS.

By **ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Miss.**

My work as published in the June-July number is the solution of the following problem :

A uniform bar, whose other dimensions may be neglected in comparison with its length ($2a$), is placed in vertical position on the edge of a platform of given height and, being slightly displaced from the vertical, is caused to revolve about the edge as an axis, the lower end being set free when the bar becomes horizontal. Where and in what manner will it strike the ground ?

The discussion of either problem—the one just stated or the one actually proposed—naturally divides itself into two parts, the first dealing with the motion before, the second after, the stick leaves the platform. It was the latter that chiefly occupied my thoughts when I prepared the solution referred to above.

I now offer the following as a discussion of the first part :

Let m=mass of stick,

$2b$=length of stick,

$2a$=other dimension perpendicular to edge of platform,

θ=angle through which stick has turned in t seconds,

ω=angular velocity at time t.

$$\frac{d^2\theta}{dt^2}=\frac{mgb\sin\theta}{m\{[(a^2+b^2)/3]+b^2\}}=\frac{3b\sin\theta}{a^2+4b^2}g;\ \omega^2=\left(\frac{d\theta}{dt}\right)^2=-\frac{6b\cos\theta}{a^2+4b^2}g+c,$$

or, since when $\theta=0$, $\omega=0$,

$$\omega^2=\frac{6bg}{a^2+4b^2}(1-\cos\theta.)\quad \text{Normal acceleration}=b\omega^2=\frac{6b^2g}{a^2+4b^2}(1-\cos\theta).$$

Component of acceleration due to gravity directly opposite to this=$g\cos\theta$.

At the instant these become equal the contact between stick and platform ceases.

This occurs when $\cos\theta=(6b^2)/(a^2+10b^2)$.

Up to this time (friction being supposed sufficiently great to prevent sliding) the stick has turned about the edge of the platform. After leaving the platform the center of gravity of the stick describes a parabola, and the stick revolves with a constant angular velocity, ω_1 about its center of gravity. This part of the motion might be discussed as in the June-July number.

DIOPHANTINE ANALYSIS.

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58. Proposed by E. S. LOOMIS, Ph. D., Professor of Mathematics in Cleveland West High School; Berea, O.

The base of a right-angled triangle is 105; find all the perpendiculars and hypotenuses to fit it, such that their values shall be integers.

I. Solution by the PROPOSER.

1. Let the hypotenuses and sides of a right-angled triangle be represented by m^2+n^2, m^2-n^2, and $2mn$.

2. Since in this problem we have to do with squares, we will draw a few observations from the squares of the integers, 1 to 18.

Int.: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18.
Sq.: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324.

3. Observations: (1) The difference between two consecutive squares is odd, the smallest difference being 3; therefore 105 is the difference between two consecutive squares; (2) The series of differences increases by 2; (3) Any difference is two times the number of the place of the lesser square in the series, $+1$, therefore let x=the number of the place of the lesser square; whence $2x+1=105$, or $x=52$, therefore all pairs of integers sought are confined to the series of squares of the integers 1 to 53; (4) The difference between squares in odd, or even, places is even; as 105, and all its factors, are odd, one must seek for differences of two squares, one occupying an odd, and the other occupying an even place.

4. Since $105=1\times105=3\times35=5\times21=7\times15=15\times7=21\times5=35\times3=105\times1$, there are eight cases to consider, each case giving 0, 1, or more than 1 pair of integers.

5. Since 105 is odd, we must have $m^2-n^2=105$.

6. Solution of cases: (1) Let $m^2-n^2=1\times105$, therefore $m=\sqrt{(105+n^2)}$. Since n is less than m, values for n, by observation (3), will be integers less than 53. Therefore by inspection, when

$$\begin{array}{rrrr} n=4, & 8, & 16, & 52, \\ m=11, & 13, & 19, & 53, \\ m^2+n^2=137, & 233, & 617, & 5313, \\ \text{and } 2mn=88, & 208, & 608, & 5512. \end{array}$$

Therefore there are four pairs of integers, for $1\times105=(11^2-4^2)=(13^2-8^2)=(19^2-16^2)=(53^2-52^2)$.

(2) Let $3(m^2-n^2)=3\times35$; whence $m=\sqrt{(35+n^2)}$; by observation (3) $n<17$, and by inspection, $105=3\times35=3(6^2-1^1)=3(18^2-17^2)$. When $n=1$, 17; $m=6$, 18; $3(m^2+n^2)=111$, 1839; and $3(2mn)=36$, 1836, giving two pairs for 3×35.

(3) Let $105=5(m^2-n^2)=5\times21=5(5^2-2^2)=5(11^2-10^2)$; whence $m=\sqrt{(21+n^2)}$. When $n=2$, 10; $m=5$, 11; $5(m^2+n^2)=145$, 1105; and $5(2mn)=100$, 1100, giving two pairs.

(4) Let $105=7\times15=7(m^2-n^2)=7(4^2-1^2)=7(8^2-7^2)$; whence $m=\sqrt{(15+n^2)}$. Therefore $7(m^2+n^2)=119$, 791; and $7(2mn)=56$, 784, giving two pairs.

(5) Let $105=15\times7=15(m^2-n^2)=15(4^2-3^2)$; whence $m=\sqrt{(7+n^2)}$, and $15(m^2+n^2)=375$, and $15(2mn)=360$, giving one pair.

(6) Let $105=21\times5=21(m^2-n^2)=21(3^2-2^2)$; whence $m=\sqrt{(5+n^2)}$, and $21(m^2+n^2)=273$, and $21(2mn)=252$, giving one pair.

(7) Let $105=35\times3=35(m^2-n^2)=35(2^2-1^2)$; whence $m=\sqrt{(3+n^2)}$, and $35(m^2+n^2)=175$, and $35(2mn)=140$, giving one pair.

(8) Let $105=105\times1=105(m^2-n^2)$. But 1 can not be the difference between two squares, by observation (1). Hence for $105=105\times1$ there is no solu-

tion. Therefore there can be but 13 pairs of integers which will satisfy the conditions of the problem, and the hypotenuses are 137, 233, 617, 5513, 111, 1839, 145, 1105, 119, 791, 375, 273, and 175; and the perpendiculars are 88, 208, 5512, 36, 1836, 100, 1100, 56, 784, 360, 252, and 140.

The principles established and employed in the above problem will hold whenever the given number is odd. But if the number is even, say 104, then from the series of squares above new observations must be drawn and employed.

II. Solution by SYLVESTER ROBBINS, North Branch, New Jersey.

Every rational right-angled triangle having 105 for its base must have an even number for its perpendicular and an odd one for its hypotenuse. Notice that of the factors entering into 105, each of these, 3, 5, 7, is difference of two squares *once*; each of the factors, 15, 21, 35, is the difference of two squares *twice*; and 105 itself is the difference of two squares *four times*. The perpendicular in each of these triangles must be twice the *product* of two roots, the base must be the difference of their squares, and the hypotenuse must be the *sum* of the same two squares. In order to obtain the *greatest* number of rational right triangles having same base, the n factors of said base must be prime numbers, never using 2, then $(3^n-1)/2$ will express the number of triangles, 1, 4, 13, 40, 121, 364, 1093, 3280, etc. In this problem there are $3\times1+3\times2+1\times4=13$ triangles. The following are general expressions for sides of rational right triangles. Roots: $r=2\sqrt{35}$, $3\sqrt{21}$, $4\sqrt{15}$, $6\sqrt{3}$, $18\sqrt{3}$, $5\sqrt{5}$, $11\sqrt{5}$, $4\sqrt{7}$, $8\sqrt{7}$, 11, 13, 19, 53; $s=1\sqrt{35}$, $2\sqrt{21}$, $3\sqrt{15}$, $1\sqrt{3}$, $17\sqrt{3}$, $2\sqrt{5}$, $10\sqrt{5}$, $1\sqrt{7}$, $7\sqrt{7}$, 4, 8, 16, 52; base $=r^2-s^2=105$ in each case; perpendiculars $=2rs=140$, 252, 360, 36, 1836, 100, 1100, 56, 784, 88, 208, 608, 5512; $h=r^2+s^2=175$, 273, 375, 111, 1839, 145, 1105, 119, 791, 137, 233, 617, 5513, being the 13 values in order, respectively.

III. Solution by JOSIAH H. DRUMMOND; LL. D., Portland, Me., and G. B. M. ZERR, A. M., Ph. D., The Russell College. Lebanon, Va.

Let h=hypotenuse, p=perpendicular. Then $h^2-p^2=(105)^2$ or $h+p=(105)^2/(h-p)$. $\therefore$ $h-p$ must be some factor of $(105)^2$; $h+p$ is also; and as the factors are all odd, we have integral values of h and p for every integral factor, and, moreover, for all factors less than 105, these values will be positive. The prime factors are 1, 3, 3, 5, 5, 7, 7; and the factors less than 105, of $h-p$, are readily found to be 1, 3, 5, 7, 9, 13, 21, 25, 35, 45, 49, 63, 75; the corresponding factors, or values of $h+p$ are 11025, 3675, 2205, 1575, 1225, 735, 525, 441, 315, 245, 225, 175, 147. Hence, $h=5513$, 1839, 1105, 791, 617, 375, 273, 233, 175, 145, 137, 119, 111; and $p=5512$, 1836, 1100, 784, 608, 360, 252, 208, 140, 100, 88, 56, 36; and $b=105$ in each case.

M. A. GRUBER, A. M., War Department, Washington, D. C., finds the 13 sets of values as given above, and refers to the full explanation of finding these results as given in his paper on "Integral Sides of Right Triangles," pages 106—108, of Vol. IV, No. 4 (April, 1897), of MONTHLY, where the problem, "Given one of the legs of a right-triangle of integral sides to find the other leg and the hypotenuse," is fully treated.

A. H. Bell, Hillsboro, Ill., finds the 13 sets of values, and deduces the general rule for finding the same from the principle shown in his solution of Problem 31, page 363, Vol. II, of Monthly. P. S. Berg, A. M., Principal of Schools, Larimore, N. D., sends a good solution finding the 13 sets of values by the method of solution No. I, of the problem last referred to.

William Hoover, A. M., Ph. D., Ohio University, Athens, Ohio, solves substantially as in Solution I, above, but as he did not carry out the work far enough to give all the results, we adopted the phraseology of the Proposer.

Also excellently solved, but without determining the full sets of results, by Otto Clayton, A. B., Remington, Ind.; J. O. Mahoney, B. E., M. Sc,. Lynnville, Tenn.; and O. W. Anthony, M. Sc., Instructor in Mathematics in Boys' High School, New York City.

AVERAGE AND PROBABILITY.

56. Proposed by B. F. FINKEL, A. M., M. Sc.,Professor of Mathematics and Physics, Drury College, Springfield, Mo.

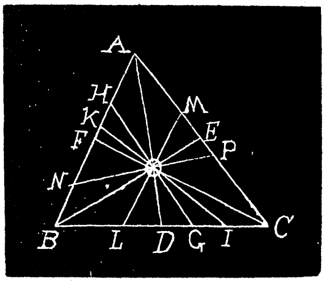
Find the chance that the center of gravity of a triangle lies inside the triangle formed by three points taken at random within the triangle. [From *Williamson's Integral Calculus.*]

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

Let D, E, and F be the middle points of the respective sides of the triangle. Then O, the common intersection of HD, BE, and CF, is the center of gravity of the triangle. Suppose that the first point is somewhere upon the line OI, and the second somewhere upon the line OG. Put $DI=x$ and $DG=y$. Then the favorable positions for the third point are upon the surface OKH, whose area is

$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}-\frac{4y}{a+9y}\right)$$

y being less than x, and x less than $\frac{1}{2}a$.



If $LD=y$, and the second point be upon OL, the favorable positions for the third point are upon the surface $OKAMO$, whose area is

$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}+\frac{4y}{a+6y}\right).$$

If $FN=y$ and the second point be upon the line ON, the favorable positions for the third point are upon the surface $OKAPO$, whose area is

$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}+1-\frac{4y}{a+6y}\right).$$

If G be taken between I and C, H will fall between F and K, and the favorable positions for the third point will be upon OHK, whose area is